ROCK FAILURE CRITERIA AND MECHANISMS OF FAILURE FOR EXCAVATION STABILITY DESIGN: A CHRONOLOGICAL EVOLUTION

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ABSTRACT

Failure of rock, when it does occur, follows definite mechanical laws and that its probability, can, at least in some instances, be predicted sufficiently in advance of actual occurrence so as to prevent any great danger to the miner. The article chronologically describes rock failure criteria of some practical and most frequently featured strength criteria for predicting failure mechanism of rocks from rock sample or models (lab scale) to total structure or rock mass breakdown (field scale). A cross section range of criteria, from the typical and indispensable to state of the art ones, categorised under isotropic or anisotropic, stress, strain and energy related failure criteria that are predominantly derived on the basis of Coulomb, von Mises and Griffith theories of failure are reviewed. More attention and discussion has been given to most practical criteria like the Mohr-Coulomb and Hoek-Brown. Evolution of mechanisms of rock failure over time by various authors. The paper demonstrates progression of rock failure criteria, and that current theories hinge on classical ones.

Keywords: Rock, Rock mass, Rock failure mechanism, Failure Criteria, Strength.

INTRODUCTION

Rock failure or rock mass failure is deviant behaviour from a specified or expected standard, although not necessarily abnormal in the natural state. Macroscopic studies, on how excavations yield in response to stresses started from way back when observations were conducted in the field and theories drawn from tests on how rockmasses fail, for example, Coulomb in Sandhu (1972) and Fayol in Brady and Brown (1985).

The purpose of this review is to bring together, and in sequence, chronologically rock failure criteria to aid understanding of rock and rock mass behaviour.

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FAILURE CRITERIA

Over the years, comprehensive laboratory studies have yielded a variety of failure criteria to describe rock strength in compression and understanding of the intact rock strength properties. Under natural conditions experiences rock different stress conditions. Fractures take place in a rock at a certain stage when it exceeds the limit stress value. The rock fails with fractures developed from the amalgamation of several micro-cracks, and diverse failure modes of rock under various stress conditions are possible. The failure process for a brittle rock under uniaxial compression can pass through four phases namely crack closure, linear elasticity, stable crack growth and unstable crack growth (Li and Nordlund, 1993; Stacey and Page, 1986).

Development of rock failure criteria can be traced way back as early as 1773 when Coulomb was involved in the research investigating retaining walls for military so that he could make taller walls than usual to stop falling. He therefore embarked on a study of lateral earth pressure acting on walls and to understand the shear strength of soils (Labuz and Zang, 2012).

A failure criterion corresponds to a yield criterion, if "failure" denotes the inception of yielding (plastic). Some of the prominent failure/strength criteria are presented as follows:

1. Tresca (1864)
2. Mohr-Coulomb (1900)
3. Von Mises (1913) and Maxwell-Huber-Hencky–von Mises theory (1924)
4. Griffith (1921)
5. Drucker-Prager hypothesis (1952)
6. Modified Griffith (1962)
7. Murrel (1964)
8. Stassi D'Alia (1967)
9. Modified Wiebols-Cook (1968)
10. Mogi (1971) and Mogi-Coulomb (2005)
11. Modified Lade (1977)
12. Hoek-Brown (1980)
13. Modified Hoek-Brown (1992)
14. The Generalized Hoek-Brown Failure Criterion (1995) updated and extended versions
15. Energy failure criteria (2009)
16. Strain Failure Criteria (2010)
17. Hoek-Brown extended (complement) version (2011) for anisotropic rock
18. Strain Energy Failure Criteria (2016)

In this section, various failure criteria are reviewed that form the basis or material of rock failure mechanism of excavations in rock.

Tresca (1864)

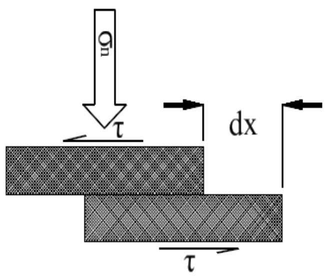
Failure is initiated when shear stress on the plane equals the critical shear stress of the material at yielding. Tresca Criterion is the simplified form criterion defining shear failure and grounded on Mohr's stress circle (Rahimi, 2014).

Mohr-Coulomb (1900)

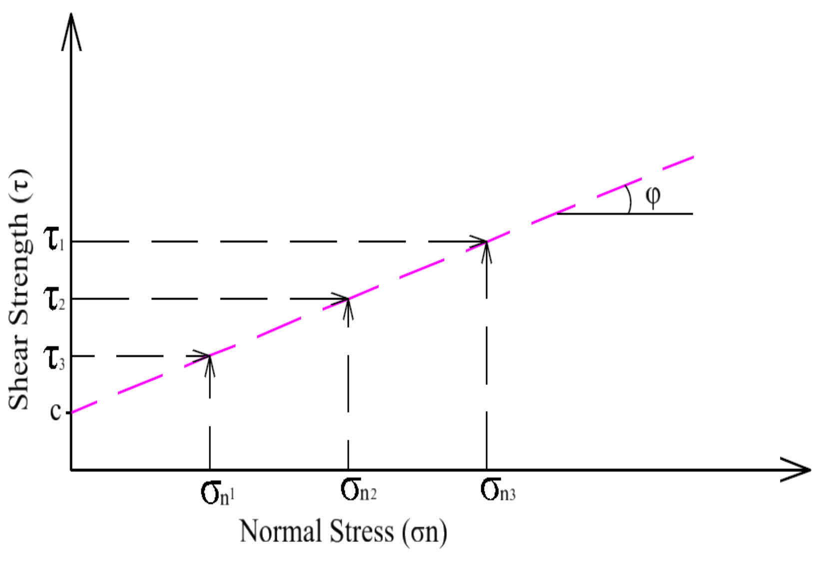
Charles-Augustin de Coulomb, through tests, developed a shear strength formulation and deduced that soil shear strength composed of parameters cohesion (c), that is independent of stress, and internal friction angle (φ), similar to sliding friction, that is dependent on stress (Sandhu, 1972; Labuz and Zang, 2012).

The shear strength test data for different normal stresses (σ), resulting in different shear stresses (τ), was plotted on a (τ-σ) axis projecting a straight line represented by the equation $\tau = c + \sigma [\tan(\varphi)]$. As portrayed by Figure 1, shear strength is directly proportional to normal stress, and failure in shear occurs when shear stress exceeds shear strength.

Figure 1: Coulomb failure criterion graphical description

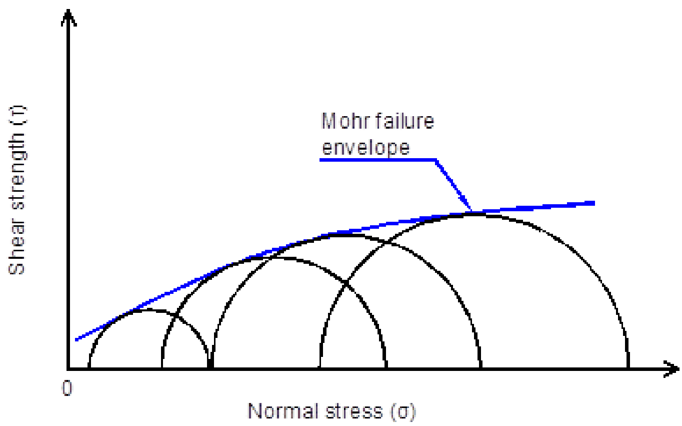


Shear failure transpires if shear strength of the formation is exceeded. The failure will not occur if the values of σ and τ lie below the strength envelope (Figures 1 and 2) (Sandhu, 1972; Labuz and Zang, 2012; Rahimi, 2014).



In 1900 Otto Mohr put forward a criterion for the shear failure of materials on a plane as a function of normal stress, $\tau = f(\sigma)$ where τ is the shear strength and σ the normal stress on that plane. Mohr proposed a Mohr failure envelope by employing the Mohr circles (Figure 2). The Mohr envelope is a line tangent to the plot of circles at different stresses and any point on the line represents failure (Sandhu, 1972; Labuz and Zang, 2012; Rice, 2010)

Figure 2: Mohr circles and envelope



The Mohr-Coulomb strength criterion proposed in 1900 by Mohr (Rahimi, 2014) is the combination of the Coulomb and Mohr Mohr theories (criteria) and the resultant failure envelope is a straight line tangent to the maximum number of Mohr circles. It states that when shear failure take-place, shear stress magnitude is related to that of the normal stress across the failure surface, the relationship being controlled by the strength of the material.

Figure 3: Mohr and coulomb combination of failure theories

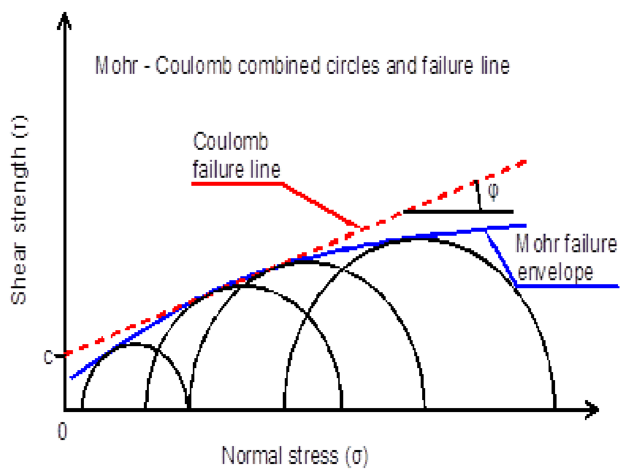
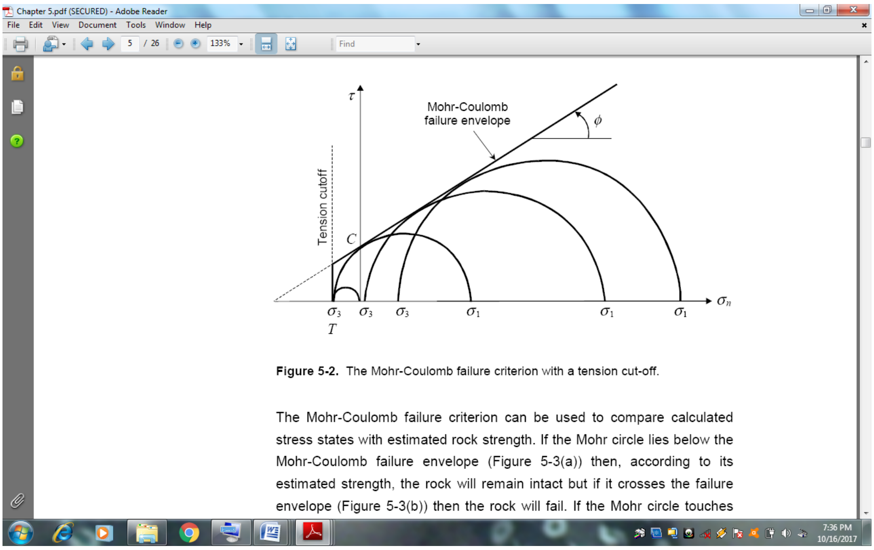


Figure 4: Mohr-Coulomb failure plotted on principal stresses



Where σ_1 - major principal stress; σ_c - Uniaxial compressive; Φ - angle of internal friction; σ_3 - minor principal stress; T - tensile strength; and c - cohesion.
Rock failure occurs if the following conditions are met:

Shear stress, $\tau = \sigma_n \tan\varphi + c$ ……… [1]

Also, $\tau = \mu\sigma_n + c$ ………. [2]

$\mu = \tan\varphi$ ……… [3]

Where σ_n *is* normal stress, c is cohesion, μ is the coefficient of internal friction angle (φ).

The tangential point of a Mohr circle (Figure 4) of principal in-situ stresses and Mohr-Coulomb failure envelope in $\tau - \sigma$ space represents state of stress in which rock will fail in shear. The Mohr-Coulomb failure criterion is also expressed in terms of maximum and minimum principal stresses (Figure 4):

$$\sigma_1 = \sigma_c + \Phi\sigma_3 \ldots\ldots\ldots\ldots \quad [4]$$

The σ_c is the uniaxial compressive strength (UCS) which is related to cohesion (c) of rock. Mohr's theory is applicable in predicting failure of brittle rockmasses.

Von Mises (1913) and Maxwell-Huber-Hencky–von Mises theory (1924)

The Von Mises criterion is a pressure-independent criterion because it defines failure mechanism which is independent of stress magnitude of material (Mises, 1913).
The criterion is based on the evaluation of the distortion energy in a given material. This is the energy that is related to deformation of the material, i.e. changes in the shape in that material in contrast to the energy which is associated with the changes in volume in the same material. Failure is initiated when the maximum distortion/shear energy in the material (rock) equals the maximum distortion/shear energy at failure in a simple tension test. If the maximum value of the distortion energy per unit volume is smaller than the distortion energy per unit volume required to cause yield in tension as tested and specified for the same material, the strength criterion states that then the structural material is stable.

He assumed that rock fails when a critical level is reached after taking a square root of the second deviatoric stress invariant (J_2).

$f(J_2) = \sqrt{J_2} - k = 0$ (Alternatively, $f(J_2) = J_2 - k^2 = 0$)………. [5]

Or $J_2 = K^2$ .... [6]

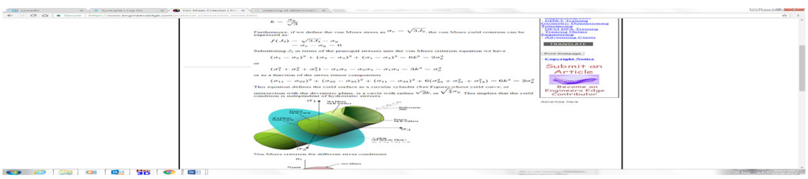
K is the yield stress of the material in pure shear.
At the onset of yielding, the magnitude of the shear yield stress in pure shear is √3 times lower than the tensile yield stress in the case of simple tension:

$\sigma_y = k\sqrt{3}$, ………. [7]

where, σ_y is the yield strength/stress of the material
At yield tensile stress, σ_v is equivalent to yield strength, σ_y in pure shear:

$\sigma_v = \sigma_y = k\sqrt{3} = \sqrt{(3J_2)}$ or $\sigma_v^{\ 2} = 3J_2 = 3k^2$ .... [8]

As a function of stress tensor components after substituting J2 in terms of principal stresses.


.... [9]

The equation defines a circular cylinder as the yield surface consisting of yield curve, or intersection with the deviatoric plane, a circle of radius $\sqrt{2}k$, or $\sqrt{2/3}\sigma_y$ implying the yield condition independent of hydrostatic stresses.

The Von Mises yield criterion proposes that the yielding of materials begins when the second deviatoric stress invariant J2 attains a critical value k. Therefore, it is also referred to as J2 plasticity. J2 flow theory developed by Hencky in 1924 defined the maximum distortion strain energy criterion as due to the fact that yield starts when the elastic energy of distortion reaches a critical value and there is a relation between J2 and the elastic strain energy of distortion (Lubarda, 1999). (Due to similarities this theory is also termed Maxwell-Huber-Hencky–von Mises theory to celebrate all of them).

Griffith (1921)

By employing the concepts of thermodynamics, Griffith (1921; 1924) studied the theory of growth of a minute crack subjected to load with the objective of expressing the uniaxial tensile strength in terms of the strain energy required to propagate micro-cracks. Griffith's fundamental view of the analysis was that as a new crack faces development and growth, some energy is dissipated.

According to the Griffith crack theory, the crack will extend if the increase in crack length has no corresponding increase in the total potential energy of the material from applied force systems (Griffith, 1921). Griffith modelled the crack extension in a plane compression criterion (Griffith, 1921):

$$(\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3) \quad [10]$$

$$\sigma_3 = -T_0 \text{ if } \sigma_1 + 3\sigma_3 < 0 \quad [11]$$

Where T_0, is the uniaxial tensile strength of the un-cracked material, σ_1 and σ_3, are vertical and horizontal stresses respectively applied to the model.

To consolidate on his criterion, Griffith (1924) utilised the theoretical model of a material consisting of a randomly oriented thin elliptical defect. The method simplified the complication to two dimensions, by ignoring synergy of bordering defects, and assuming an elastic homogenous material. It was proved that extreme tensile stresses exist at the edge of appropriately oriented thin ellipses under compressive stress conditions. It was simulated that fracture initiates from the perimeter of an open defect when the tensile stress on this perimeter exceeds the localised tensile strength of the material. The mechanism of failure is subjected to the formation, propagation, and linking of microscopic fractures with stress concentration on the front edges (a defect with a sharp tip greatly concentrates stress). When the critically oriented fractures are fully propagated and critical stress, characteristic of the material is reached, failure happens at once.

Drucker-Prager hypothesis (1952)

Drucker and Prager (Drucker and Prager 1952; Rahimi, 2014), proposed their failure criterion with the objective of extending application of Von Mises failure criterion to rock mechanics. The Drucker–Prager failure criterion is a mathematical model in three dimensions which depends on pressure to estimate the state of stress at a point of ultimate rock strength. The criterion assumes that the octahedral shear stress at failure is linearly proportional to the octahedral normal stress via material constants. Originally developed for soil mechanics and as a Von Mises criterion extension, Drucker and Prager proposed new failure criterion by including the mean normal stress component. It is a criterion that depends on pressure to determine whether a material has plastically yielded or failed. The Drucker-Prager criterion (Drucker and Prager 1952) was coined to be applied on materials differing in tensile and compressive strengths, particularly in the study of plastic deformation of soils, and can be expressed as:

$$\sqrt{J_2} = \lambda I'_1 + k \quad \ldots\ldots\ldots \quad [12]$$

Where λ and k are material constants, J2 is the second invariant of the stress deviator tensor and I'_1 is the first invariant of the stress tensor:

$$I'_1=\sigma'_1+ \sigma'_2+ \sigma'_3 \ldots\ldots\ldots \quad [13]$$

$$J_2=1/6[(\sigma'_1- \sigma'_2)^2 + (\sigma'_1- \sigma'_3)^2 + (\sigma'_3- \sigma'_1)^2]\ldots\ldots\ldots \quad [14]$$

Where, σ'_1, σ'_2 and σ'_3 are the principal effective stresses.

In terms of octahedral shear stress, τ_{oct} and octahedral normal stress, σ_{oct}':

$$\tau_{oct}= \sqrt{2/3}(3\lambda\, \sigma'_{oct} + k \ldots\ldots\ldots \quad [15]$$

Where $\sigma_{oct}'= (1/3)\, I'_1$ and $\tau_{oct}=\sqrt{(2/3 J_2)}$

Modified Griffith (1962)

Walsh and Brace (Walsh and Brace, 1964), modified the Griffith's crack theory by including friction between faces of cracks to understand the effect of friction and to account for the influences of crack closure in compression, when shear failure as a result of closure of cracks may happen before tensile stress attains a critical level at the tip of crack to initiate fracture. The criterion was developed to comprise shear mechanism and frictional behaviour. Modified Griffith theory is an extension to handle brittle fracture of anisotropic rock. (Walsh and Brace, 1964; *see* Rahimi, 2014)

Expressions of the Modified Griffith theory in principal stresses terms (Jaeger and Cook 1976), and in terms of shear and normal stresses are respectively as follows:

$$\sigma_1[(\mu^2+1)^{0.5}-\mu]-\sigma_3[(\mu^2+1)^{0.5}+\mu] = 4\sigma_t \quad \ldots\ldots\ldots \quad [16]$$

$$\tau^2+4\sigma_t.\sigma_n - 4\sigma_t^2 = 0, \text{ for } \sigma_n < 0 \quad \ldots\ldots\ldots \quad [17]$$

$$\tau=2\sigma_t+\mu\sigma_n, \text{ for } \sigma_n > 0 \quad \ldots\ldots\ldots \quad [18]$$

Where μ is the coefficient of internal friction, σ_t is tensile strength of material, σ_n is normal stress and τ is shear stress.

Murrel (1964)

The Extended Griffith criterion was introduced by Murrel (Murrell, 1964; Rahimi, 2014) and includes intermediate principal stress as a contributing factor in the strength of rock. Murrel criterion failure surface in the three dimensional stress domain predicts a circular shape cross section.

$$(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = 24T_0(\sigma_1+\sigma_2+\sigma_3)\ldots\ldots\ldots \qquad [19]$$

Stassi D'Alia (1967)

By modifying the plastification condition, Stassi d'Alia (1967) (also Rahimi, 2014) developed a non-linear failure criterion in terms of principal stresses and uniaxial tensile strength. Stassi D'Allia's criterion takes into account characteristic loading paths. The parabolic failure criterion states that failure occurs when the distortional strain energy density reaches a limiting value (Stassi-D'Alia, 1967):

$$(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = 2(C_0-T_0)(\sigma_1+\sigma_2+\sigma_3)+2C_0T_0\ldots\ldots\ldots \qquad [20]$$

c_o is the uniaxial compressive strength.

Modified Wiebols-Cook (1968)

Wiebols and Cook (Wiebols and Cook, 1968), proposed a model which describes the influence of the intermediate principal stress (σ_2) on rock strength. By micro-mechanical analysis of sliding cracks, Wiebols and Cook concluded that rock fails when shear strain energy which is enclosed within micro-cracks has hit a critical level. A model called Modified Wiebols- Cook was presented later with the mean normal stress in form of a quadratic (Rahimi, 2014).

$$\sigma_n^{0.5} = A+B\sigma_m+C\sigma_m^{\ 2} \qquad \ldots\ldots\ldots \qquad [21]$$

$$\sigma_m = (1/3)(\sigma1+\sigma2+\sigma3) \qquad \ldots\ldots\ldots \qquad [22]$$

$$\sigma_n^{0.5} = \surd[1/6\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_2 - \sigma_1)^2\}] \qquad \ldots\ldots\ldots \qquad [23]$$

$$= (3/2)^{0.5}\, \tau_{oct} \qquad \ldots\ldots\ldots \qquad [24]$$

Where σ_m is the mean effective confining stress, τ_{oct} is the octahedral shear stress and A, B and C are constants related to cohesion and internal friction angle.

Mogi-Coulomb (1971) and later extension by Al-Ajmi-Zimmerman (2005, 2006, 2007)

Mogi (Mogi, 1971; Mogi, 2007; Kwaśniewski, 2013; Rahimi, 2014) performed true triaxial compression experiments on different rock types and interpreted the results. Mogi concluded that the intermediate principal stress (σ_2) influences the rock strength for various rock lithology and the fracture occurs in the direction of intermediate principal stress along a plane, therefore the mean normal stress which opposes creation of fracture is $\sigma_{m,2}$, rather than the octahedral normal stress, σ_{oct}. Mogi further formulated the assumption that, distortional strain energy as a frictional force is proportional to the octahedral shear stress, τ_{oct} , and will be increased by increasing $\sigma_{m,2}$ up to a critical level where failure occur, leading to a theory called Mogi Failure Criterion:

$$\tau_{oct} = f(\sigma m,2) \quad \ldots\ldots\ldots\ldots \quad [25]$$

$$\sigma_m = (\sigma_1+\sigma_2)/2 \quad \ldots\ldots\ldots\ldots \quad [26]$$

$$\tau_{oct} = 1/3\sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad \ldots\ldots\ldots\ldots \quad [27]$$

Based on this result, Al-Ajmi and Zimmerman (2005) found a linear relation that could fit well with poly-axial tests data in τ_{oct} - $\sigma_{m,2}$ space:

$$\tau_{oct} = \alpha + b\sigma_m \quad \ldots\ldots\ldots \quad [28]$$

Where parameters, a is the intersection of line on the τoct axis and b is the line inclination.

The linear version of the Mogi criterion is what is known as the "Mogi-Coulomb" failure criterion.

Al-Ajmi and Zimmerman (2005, 2006, and 2007), tested the developed linear function for the poly-axial test data from eight different rock types and the results indicated that Mogi-Coulomb criterion gives a good fit line to poly-axial tests data. They also found out that this poly-axial criterion correlates well with conventional triaxial test data and therefore this linear Mogi-Coulomb failure criterion is found to be equivalent to Mohr-Coulomb in standard triaxial stress state.

Modified Lade (1977)

From observations in experiments, Lade (1977) concluded that for cohesionless soil, the frictional angle decreases as the magnitude of mean normal stress increases ((Lade, 1977; Rahimi, 2014). Ewy (1999) developed The Modified Lade criterion. In Ewy's version of Lade criterion, the material constant has been considered as zero for the purpose of modifying the original Lade criterion in the logic that linear shear strength increases with the first stress invariant increase, or mean normal stress invariant.

An invariant is a constant that is independent of orientation (θ) when the normal stresses on two or three perpendicular planes are summed up in stress tensors.

Hoek-Brown (1980)

Hoek and Brown (1980), developed an empirical failure model for fractured rock. To investigate the failure of fractured rocks, both fracture properties and rock properties should be taken into account. The failure criterion which has been proposed by Hoek and Brown included both rock and fracture properties:

$$\sigma_1 = \sigma_3 + \sqrt{[m\sigma_c\sigma_3 + s\sigma_c^2]} \quad \ldots\ldots\ldots \quad [29]$$

Where the uniaxial compressive strength, σ_c, is function of rock properties while m and s are constants subject to both rock properties and fracture characteristics. See also equation twenty-eight.

Modified Hoek-Brown (1992)

The Modified Hoek-Brown (1992) criterion satisfies the condition of zero tensile strength for jointed rock (Hoek *et al*, 1992).

$$\sigma_1 = \sigma_3 + \sigma c\{mb(\sigma 3/\sigma_c)\}^a \quad \ldots\ldots\ldots \quad [30]$$

Where σ_1 is the major principal effective stress at failure, σ_3 is the minor principal effective stress at failure, σ_c is the uniaxial compressive strength, m_b is the value of constant m for broken rock, and a is the constant for broken rock

In this version (Hoek et al. 1992), parameter mb is defined by the ratio mb/mi where mi is the material constants for intact rock.

The Generalised Hoek-Brown Failure Criterion (1995, updated in 2002), extended by Benz and Zhang (2008)

The Generalised Hoek-Brown Failure Criterion was re-evaluated by Hoek and others (Hoek et al. 2002; Hoek and Brown 1997) based on the analysis of a huge number of lab tests and the empirical model has improved by refining the constants and other input parameters estimation. Geological strength index (GSI) and disturbance (D) are introduced in the calculation of *m*, *s* and *a*.

A general form of Hoek-Brown failure criterion and refined through latest lab data in 2002 (Hoek et al. 2002) is:

$$\sigma_1 = \sigma_3 + \sigma c\{mb(\sigma 3/\sigma_c) + s\}^a \quad \ldots\ldots\ldots \quad [31]$$

Where σ_1 is the major principal effective stress at failure, σ_3 is the minor principal effective stress at failure, σ_c is the uniaxial compressive strength of intact portion of the rock mass, m_b and s are constants of rockmasss. In a scenario where a = 0.5 and constant m_b tends to m (i.e. m = mb), the criterion assumes its original form where constants s defines structural pattern or quality of rockmass and m specifies the rock type.

Benz *et al.* (2008), suggested another form of extending the Hoek-Brown criterion with intrinsic material factorisation, and further improvement by considering the influence of intermediate principal stress on failure experimentally, the factor which is ignored in the original Hoek-Brown criterion. Zhang (2008), also stretched the Hoek-Brown criterion by taking into account the effect of the intermediate principal stress component, developing failure criterion called Three Dimensional (3D) Version of the Hoek–Brown strength criterion for intact rock.

Energy failure criteria (2009)

Xie *et al.* (2009), as well as Ismael *et al.* (2015), propounded failure criteria called 'energy criterion' applicable to rocks. The criterion is hinged on elastic energy and the stress deviator, in which some elastic strain energy and energy limit values are compared to balance out the energy supplied to grow a crack and the amount of energy dissipated due to the formation of new surfaces and other processes such as plasticity that dissipates energy.

In this theory rock material deformation due to an external load is considered as a closed system. Therefore, the assumptions are that there is no heat conversion from mechanical work and that the total energy created as a result of work done by the external load may be evaluated according to the first law of thermodynamics: Total energy is equal to sum of dissipated energy and releasable elastic strain energy stored in a rock.

The dissipated energy causes internal damage and irreversible plastic deformation in rock.

Energy dissipation continually damages the internal microscopic structure of rock, which leads to a deterioration in rock strength and eventually complete structural failure. Hao and Liang (2016), advanced a strength criterion established on shear strain energy on the failure plane.

Strain Failure Criteria (2010)

In strain failure criteria, failure spreads when strain energy release rate surpasses a strain limit value called critical strain energy release rate.

Kwaśniewski and Takahashi (2010) (*also* Ismael *et al.* 2015), summed up strain based criteria for rocks. They are founded either on maximum principal strain (tension), maximum principal strain (compression), maximum shear strain or mean and octahedral strain.

Hoek-Brown extended (complement) version (2011) for anisotropic rock

In the original and modified forms of Hoek-Brown criterion it is simply assumed that the behaviour of intact or the rock mass is isotropic and therefore parameters **m** and **s** for such a rock are evaluated. Hoek and Brown (1980), initially presented reduction factors modifying m and s that are special for anisotropic rock. Bagheripour and Mostyn

(1996) evaluated this idea through a model for a soft sedimentary rock. Bagheripour and Hakimipour (2009) examined the Hoek-Brown failure criterion efficiency when applied for prediction of rock mass strength to dam foundations and tunnels. Wu and Zhou (2010) employed a relatively large number of test data proposed and evaluated a new model based on Hoek-Brown failure criterion for predicting strength of columns confined with polymers reinforced by fibre.

Saroglou and Tsiambaos (2008) and Bagheripour *et al.* (2011), proposed expressions that take into account anisotropy in predicting rock strength. This provides the best alternative and solution for predicting anisotropic strengths. The product of the special reduction factor or decay function and the original form of Hoek-Brown criterion for truly intact rock, results in strength of anisotropic rock. The decay function accounts for relative reduction of strength due to existence of planes of weaknesses. Therefore the complement functions as a multiplier for deriving strength variation of anisotropic rock as a varying fraction of the corresponding intact rock. This advanced compliment is generally applicable to all failure criteria evaluating the intact rock strength.

Strain Energy Failure Criteria (2016)

Chen *et al.* (2016), (*also* Ismael *et al.* 2015) concluded a strength criterion similar to Hoek and Brown criteria based on strain energy. The parameters for the three constants are obtained by curve fitting. For a basic version, the parameters are reduced to two.

CONCLUSION

The paper has reviewed progression of rock failure criteria and mechanisms of rock and rockmass failure around mining excavations, and the associated intricate relationships with major, intermediate and minor stresses present in the rockmass.

Failure theory is the science of predicting the conditions under which solid materials fail under the action of external loads. Rock failure is the loss of load carrying capacity of the rockmass. The laboratory formulation of the failure strength criteria elucidated in the foregoing has helped explain the failure mechanism of rockmass in excavations on field scale, as follows.

1. The expected failure mechanism in excavation can easily be predicted by numerical modelling by the application of the failure criteria. The failure criteria are used to simulate the path that describes the stages the excavation rockmass undergoes before collapse when stressed. Rocks deform or strain when stressed. The change in size, shape, volume or displacement or translation along the break of material is what is referred as strain. There are three stages of deformation that a rock subjected to increasing stress undergoes.

(a) Elastic deformation in which the strain is recoverable or reversible;

(b) Plastic (ductile) where the strain is irrecoverable or irreversible; and

(c) Rupture (fracture) in which irreversible strains breaks the material.

2. Rockmass failure is a complex process due its nature of being a non-ideal homogeneous, isotropic and elastic solid. Heterogeneity, anisotropy, confinement, block geometry, boundaries and other pre-existing defects affect the strength, mechanisms and modes of failure that develop according to strength properties, heterogeneity, anisotropy and degree of disturbance. Every material under stress will deform. The deformation is governed by the constitutive behaviour of the material.

Depending on the behaviour of the material under stress, two classes can be identified Brittle as materials and Duchile materials failure.

Brittle materials (with brittle failure fracturing) have a small or large region of elastic behavior but only a small region of ductile (plastic) behaviour before they fracture. In brittle behaviour, rocks exhibit elastic behaviour followed by rupture. Failure point is reached when a brittle rock loses all resistance to stress and crumbles, or experiences increased deformation.

Brittle rock failure is determined by various criteria types:

1. Stress-strain criteria where the material fails when maximum principal stress or strain, $\sigma1$ or $\varepsilon1$, is reached exceeding uniaxial tensile stress or strain, σt or εt, or when the minimum principal stress or strain, $\sigma3$ or $\varepsilon3$ is surpassed by uniaxial compressive strength or strain, σc or εc.

This is the principle of failure behind Tresca, von Mises, Mohr-Coulomb, Drucker-Prager, Hoek-Brown, Mogi-Coulomb and Modified Lade failure criteria

2. Linear elastic fracture mechanics determines failure by estimating the amount of energy needed to develop a pre-existing crack. Griffiths' and Murrel theory falls under this category.
3. Energy-based methods are suitable for anisotropic or complex geometry materials and an approach called strain energy release rate is useful. Strain energy can be obtained from the stress-strain plots by calculating the area under the graph. Stassi D'Alia and Modified Wiebols-Cook are examples.

Ductile materials failure (yielding) criteria are predicted by yield criteria such as the Tresca and the Von Mises yield criteria. Ductile materials have a small region of elastic behavior and a large region of ductile (plastic) behavior before they fracture. In ductile behaviour, rock experience large amounts of plastic deformation before rupture

The prevailing conditions such as temperature, state of stress, loading rate and mechanical properties determine whether materials fail in a brittle or ductile manner or combination.

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